An inverse optimal control approach to human motion modeling

Katja Mombaur, Jean-Paul Laumond and Anh Truong

Abstract In this paper, we present inverse optimal control as a promising approach to transfer biological motions to humanoid robots. Inverse optimal control serves to identify the underlying optimality criteria of human motions from measurements. Based on these results optimal control models are established that can be used to control robot motion. Inverse optimal control problems are hard to solve since they require the simultaneous treatment of a parameter identification problem and an optimal control problem. We propose a bilevel approach to solve inverse optimal control problems which efficiently combines a direct multiple shooting technique for the optimal control problem solution with a derivative free trust region optimization technique to guarantee the match between optimal control problem solution and measurements. We apply inverse optimal control to determine optimality principles of human locomotion path generation to given target positions and orientations, using new motion capture data of human subjects. We show how the established optimal control model can be used to enable the humanoid robot HRP-2 to autonomously generate natural locomotion paths.

1 Introduction

1.1 Inverse optimal control: what is the optimization criterion of human motion?

It is a very common assumption in bionics and biomechanics that natural structures and processes are optimal. This is also true for many forms of human and animal motion such as locomotion [Alexander (1984), Alexander (1996)]. However,

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the specific optimization criterion applied to a particular motion is very often not known. But is is possible to observe the results of this natural optimization process by measurements, such as motion capture, EMG etc.

From a mathematical perspective, the generation of motions of animals and humans can be formulated as optimal control problem. Optimal control problems are a special type of optimization problems where the unknown variables are not represented by a simple *n*-dimensional vector, but by *n* unknown functions in time, more specifically, unknown input or control functions, and unknown state functions. A dynamical model establishing the relationship between control and state functions represents a constraint of the optimal control problem. The objective function (which is also calles the cost function) may depend on both, control and state functions, as well as on time. For a classical (forward) optimal control problem the full problem formulation including objective function, model etc. is known and the solution has to be determined.

But as stated above, we are often facing the opposite problem, namely that the exact objective function is not known, but instead we know the solution to this problem, or at least its observable part, from measurements. This type of problem is called an inverse optimal control problem (compare fig. 1). These problems are much harder to solve than (forward) optimal control problems. Inverse optimal control problems are also much more difficult than standard identification problems since optimization and data fitting have to be handled simultaneously.

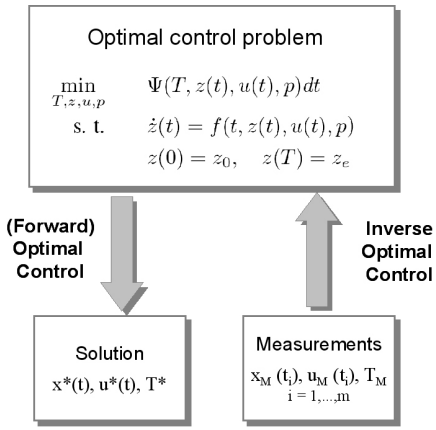

Fig. 1 (Forward) Optimal control problems vs. Inverse optimal control problems

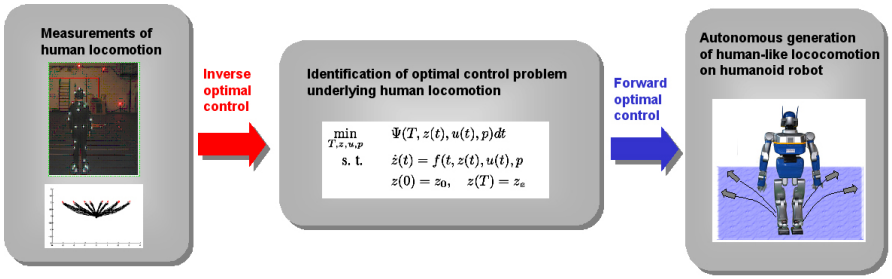

Fig. 2 The inverse optimal control approach helps (a) to understand optimality of human locomotion and (b) to generate natural humanoid locomotion

1.2 Natural humanoid locomotion by inverse optimal control

We consider the understanding of the optimality principles of human locomotion as one of the keys to generate biologically inspired locomotion on autonomous robots. In fig. 2, we give an overview of the inverse optimal control approach that we propose in this paper. It basically consists of three steps: (a) identification of human optimality criteria for locomotion by inverse optimal control from motion capture measurements, (b) formulation of the full (forward) optimal control model, and (c) implementation and solution of optimal control problem on humanoid robot. This approach enables a humanoid robot to autonomously generate its natural locomotion trajectory to any requested target.

Human and humanoid locomotion can be investigated on different levels. Most research on humanoid robot locomotion aims at generating trajectories on the joint level. The straight or bent path on the floor along which the humanoid robot is supposed to move, is prescribed for this purpose. The study of the selection and optimal generation of this overall path has however been widely neglected in humanoid robotics so far. If humans are asked to walk towards a given end position and orientation in an empty space with no obstacles, they will select a very specific path, out of an infinite number of possibilities. In the attempt to control humanoids in a biologically inspired manner, it would be desirable to understand and imitate that behavior of humans.

In this paper, we show how the inverse optimal control approach is used to generate natural overall locomotion trajectories from an initial rest position and orientation to a given target rest position and orientation. For this purpose, we are not interested in studying the individual trajectories of all joints. Instead, the locomotor system can be described by its overall position and orientation in the plane.

However, inverse optimal control problems are prevalent and can basically be found everywhere in natural sciences, and the proposed approach is very general. Consequently, the approach can also be used to analyze motions on joint level.

1.3 Related Work

Classical imitation problems have been widely studied for humanoid robots, leading to impressive results (e.g. [Nakazawa et al (2002)], [Ikeuchi (2009)], [Suleiman et al (2008)] and [Billard and Mataric (2001)]). The task here consists in reproducing a human movement within the kinematic and dynamic ranges of a robot, using different approaches for model identification, such as learning techniques or optimization, but the underlying optimality principles of the motions are not at all investigated. However, the simultaneous treatment of an imitation problem and an identification of optimality principles - i.e. the "inverse optimal control problem" discussed above - is much more difficult and has not yet been extensively investigated.

[Liu et al (2005)] present a realistic generation of character motion by physics-based models. In their case, the objective function is assumed to be known (minimization of joint torques), but they identify other unknown model parameters from measurement sequences by a nonlinear inverse optimization technique. Heuberger provides a detailed overview of inverse optimization for the different class of combinatorial problems [Heuberger (2004)]. Inverse optimal control problems formulated as bilevel problems can also be treated as MPEC (Mathematical programs with equilibrium constraints). Here the optimal control problem is replaced by the corresponding first order optimality conditions which become constraints of the parameter estimation problem (see the book [Luo et al (1996)]). There is theoretical research on MPECs and corresponding optimality conditions and constraints qualifications (e.g. [Ye (2005)]) but the approach is very difficult to implement in practice. In a recent thesis, it has been applied to very simple dynamical models [Hatz (2008)].

For mobile wheeled robots, the problem of generating the overall path (i.e. the trace on the floor) has been extensively studied (see e.g. [Latombe (1991)], [Laumond (1998)], [LaValle (2006)]). The focus here was on finding a feasible - not an optimal - path in the presence of many obstacles, and no biological inspiration was required in this case. A mobile robot is generally performing nonholonomic movements, i.e. the direction of motion depends on the orientation of the robot or of its wheels.

In humanoid robot research, several authors have studied real time path planning and adaption based on sensor information, looking at the same time at the shape of the path and an appropriate choice of footholds (e.g. [Stasse et al (2006), Chestnutt et al (2005), Gutmann et al (2005), Yoshida et al (2008)]). The problem of natural off-line locomotion path planning has not yet received much attention. In [Mombaur et al (2008)], we have proposed a heuristic optimal control model

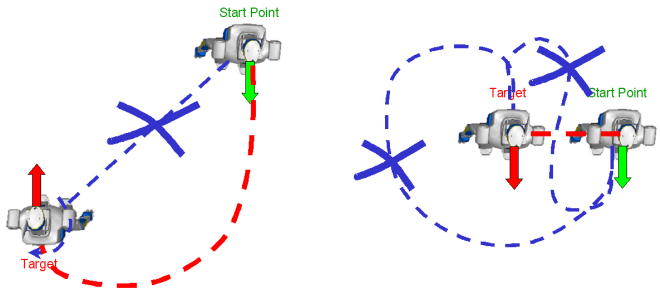

Fig. 3 Natural locomotion paths for humanoid robots: Examples of realistic and unrealistic paths (red dashed lines vs. blue dashed lines) for two different targets

to autonomously generate naturally shaped locomotion paths for humanoid robots. [Choi et al (2003), Pettré et al (2003), Brogan and Johnson (2003)] have studied off-line planning for biped locomotion in computer graphics.

From a biological perspective, the shape of human locomotion paths has been investigated, e.g. [Hicheur et al (2007)]. In particular it has been shown that human locomotion in many cases is nonholonomic just as wheeled motion, i.e. people tend to move in forward direction rather than sidewards ([Arechavaleta et al (2008b), Arechavaleta et al (2008a), Laumond et al (2007)]. This general preference may easily be understood from the human anatomy. On the other hand, there are certain situations in which humans naturally tend to abandon the nonholonomic behavior and to include sideward or oblique steps in the locomotion, i.e. move in a holonomic way. This obviously occurs when obstacles must be avoided, but also in the case of very close goals without obstacles (compare fig. 3, right part). In [Mombaur et al (2008)], we have made a first attempt to establish a model that continuously selects between holonomic and nonholonomic locomotion in a realistic way.

1.4 Contribution of this article

The first contribution of this paper is to propose inverse optimal control as a general approach to transfer biological motions to robots. Inverse optimal control not only helps to understand the underlying optimization objectives of recorded biological motion. It also leads to the generation of mathematical forward optimal control models that can be applied to control humanoid robot motions in a natural way. In this paper, we describe the general form of inverse optimal control problems as well as a very flexible numerical technique for their solution.

The second contribution of this article is to present an example of a successful application of inverse optimal control: a unique optimal control model of the overall locomotion path generation to close targets - i.e. to given final position and orientation, while zero speed is requested at start and end time. In our previously mentioned

research [Mombaur et al (2008)], a qualitative model was created and parameters were selected by manual tuning. In contrast to this, the goal in the present paper was to truly identify the weights of the proposed optimal control model from human motion capture data. In addition, based on the inverse optimal control results, we could even simplify the previous formulation by establishing a unique model with constant weight factors that is valid for a whole domain of targets.

This paper is organized as follows: In section 2, we present the general inverse optimal control problem formulation as well as a general numerical solution technique. In section 3, we show how inverse optimal control has been applied to generate natural human-like locomotion paths by outlining the whole sequence from motion capture experiments over model identification to implementation on the humanoid robot HRP-2. In the final section, we summarize results and discuss future research.

2 Inverse optimal control: A general approach to understand natural processes

The goal of inverse dynamic problems is to determine the formulation of an optimal control problem - and in particular its cost function - that is able to best reproduce the available experimental data. In this section, we present the problem statement of inverse optimal control problems, as well as a newly developed numerical solution technique.

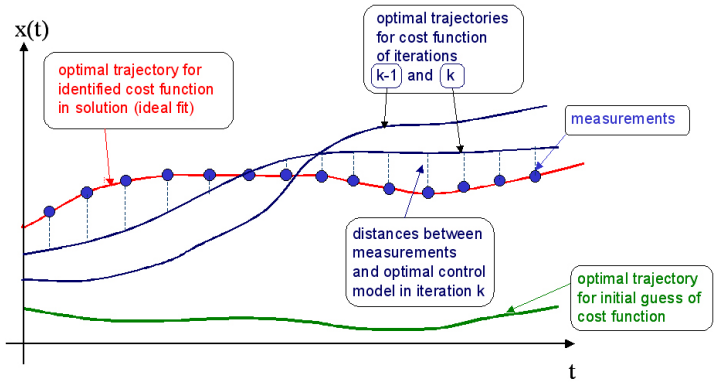

Fig. 4 Goal of inverse optimal control: identify cost function that best approximates measured data

2.1 Formulation of inverse optimal control problem

An inverse optimal control problem consists in determining the function $\Phi(x(t),u(t))$ in the objective function (1) of the following optimal control problem

$$\min_{x(\cdot),u(\cdot),T} \int_0^T \Phi(x(t),u(t))dt \tag{1}$$

$$\text{s. t.} \quad \dot{x} = f(t,x(t),u(t)) \tag{2}$$

$$x(0) = x_0, \quad x(T) = x_e \tag{3}$$

when its solution is known. $x(t)$ are the state variables and $u(t)$ the control variables. The dynamic model (2), and initial and final conditions (3) are assumed to be known. We assume that the solution $x^*(t) \in \mathbb{R}^{n_x}$, $u^*(t) \in \mathbb{R}^{n_u}$ is not known continuously, but only at m evenly space points. In many practical cases, not the full solution is observable, and only some components of the optimal states and controls $x^*_{red}(t) \in \mathbb{R}^{n_{xr}}$ and $u^*_{red} \in \mathbb{R}^{n_{ur}}$ (where $0 < n_{xr} < n_x$ and $0 < n_{ur} < n_u$) are known at m discrete points.

The inverse optimal control problem consists in determining the exact objective function $\Phi(\cdot)$ that produces the best fit to the measurements in the least squares sense (compare fig. 4). We make the basic assumption that the objective function can be expressed as a weighted sum of a series of base functions $\phi_i(t)$ with corresponding weight parameters α_i:

$$\Phi(x(t),u(t),\alpha) = \sum_{i=1}^{n} \left[\alpha_i \int_0^T \phi_i(x(t),u(t))dt \right] \tag{4}$$

The problem of determining the objective function $\Phi(\cdot)$ resulting in the best approximation thus is transformed into the problem of determining the best weight factors α_i.

The base functions $\phi_i(x(t),u(t))$ describe reasonable potential components of the objective function in the given situation. It is important to choose a non-redundant set of objective functions since different base functions leading to exactly the same behavior would be impossible to identify.

In a combined objective function only the relative and not the absolute size of the weight factors counts. If a weight α_i is large, the corresponding term has a big effect on the overall sum and therefore is more likely to be reduced in the overall context. If α_i is small (in the extreme case zero) the term has little (or no) influence on the objective function and the quantities can become large without doing much harm.

With this parameterization of the objective function (4), we can formulate the inverse optimal control problem as bilevel problem:

$$\min_{\alpha} \sum_{j=1}^{m} ||z^*(t_j;\alpha) - z_M(t_j)||^2 \tag{5}$$

where $z^*(t;\alpha)$ is the solution of

$$\min_{x,u,T} \int_0^T \left[\sum_{i=1}^{n} \alpha_i \phi_i(x(t),u(t)) \right] dt \tag{6}$$

$$\text{s. t.} \quad \dot{x} = f(t,x(t),u(t)) \tag{7}$$

$$x(0) = x_0, \quad x(T) = x_e \tag{8}$$

The vector z stands for the full or reduced vector of states and controls, $z(t)^T = (x(t)^T, u(t)^T)$ or $z(t)^T = (x_{red}(t)^T, u_{red}(t)^T)$, depending on the case treated, i.e. the available measurements. z_M denotes the measured values.

2.2 Numerical solution of inverse optimal control problems

In this section we present a pragmatic numerical approach to the solution of inverse optimal control problems treated as bilevel problems (compare fig. 5). The upper level handles the iteration over the objective function parameters α such that the fit between measurements and optimal control problem solution is improved. Each upper level iteration includes one call to the lower level where a forward optimal control problem is solved for the current set of α_i. The optimal solution of this problem is then communicated back to the upper level such that the least squares fit between measurements and computations can be evaluated.

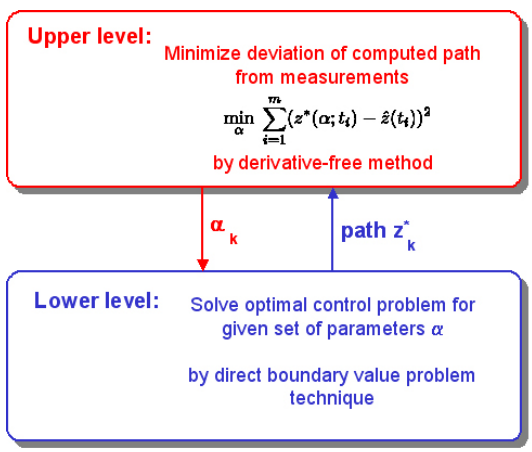

Fig. 5 Solution of inverse optimal control problem as bilevel optimization problem

As described in [Mombaur (2009)], we have implemented and tested a method to solve inverse optimal control problems on the basis of two powerful numerical techniques. We propose a combination of efficient direct techniques for the solution of the lower level optimal control problem, and of an efficient derivative-free method for the solution of the upper-level least-squares problem. Both techniques that we have combined in our modular software environment will be briefly described in this section.

For the solution of the lower level optimal control problem we have applied the highly efficient direct boundary value problem approach using multiple shooting developed by Bock and co-workers (MUSCOD [Bock and Plitt (1984)] [Leineweber et al (2003)]). The MUSCOD method uses a direct approach (also called a first-discretize-then-optimize approach) to handle control functions. State functions are treated by a multiple shooting technique which transforms the original boundary value problem into a set of initial value problems with corresponding continuity and boundary conditions. The resulting structured nonlinear programming problem (NLP) is solved by a tailored sequential quadratic programming (SQP) algorithm. It is important to note that this approach still includes a simulation of the full problem dynamics on each of the multiple shooting intervals. This is performed simultaneously to the NLP solution using fast and reliable integrators also capable of an efficient and accurate computation of trajectory sensitivity information [Bock (1987)].

For the solution of the upper-level least squares problem, we apply a derivative-free optimization technique, i.e. it only requires function evaluations and does not need derivatives. Derivative-free optimization is always favorable if function evaluations are expensive and noisy and derivative information can therefore not be generated in a reliable manner. In the case of our bilevel problem, each function evaluation of the upper-level problem corresponds to a solution of the lower-level optimal control problem, so it would definitely be difficult to generate numerical derivatives of this function. We only have to handle simple box constraints on the weight parameters in the upper level, all other constraints are handled by the optimal control code in the lower level. We use the newly released derivative-free optimization code BOBYQA [Powell (2008)], which is a very efficient derivative-free optimization technique. It is an extension of Powell's well known code NEWUOA, and can additionally handle simple bounds on the variables. Interpolation-based trust region techniques of derivative-free optimization are used to establish a quadratic polynomial model of the objective function, based on function evaluations only.

As state above, in a combined objective function of an optimization problem only the relative size of parameters matters, not the absolute size. Consequently, the identification of parameters by inverse optimal control is therefore only possible up to a common constant. Our practical way to tackle this issue is to fix one of the parameters a priori to 1.0 and to determine the remaining parameters. If by mistake a parameter that actually should be zero in the solution has been fixed to a nonzero value, a strange behavior of the numerical iterations will be observed, and computations should be repeated with a different parameter fixed.

3 Application of inverse optimal control to study human locomotion

The purpose of this section is to demonstrate how inverse optimal control has been successfully applied to determine optimality criteria of human locomotion. It also describes how this optimal behavior is implemented on a humanoid robot. We are interested in the shape and temporal development of the overall locomotion trajectories, i.e. the traces of the human locomotion on the floor, for given start and end positions and orientations.

3.1 Experiments: human locomotion trajectories

We have performed a series of experiments to capture human locomotion trajectories for given start and end positions and orientations and with zero initial and final speed, in particular to close-by targets in a radius of ~ 3.5 m.

Ten healthy male subjects participated in the experiments, with an average height of $1.77 + / - 0.06$ m and an average age of $27 + / - 4$ years. All subjects gave their informed consent to perform the experiments. We have used a Motion Analysis motion capture system with 10 cameras, all with a sampling frequency of 100 Hz.

100 different target scenarios, i.e. different combinations of target positions and orientations, were selected, and randomly ordered, using each scenario twice, resulting in 200 motions preformed by each subject. An arrow on the floor indicated target position and orientation of each trial. The experimental setup is shown in fig. 6.

We were interested in recording the time histories of the overall positions $x(t)$ and $y(t)$ and orientations $\theta(t)$ of the subjects. This could be achieved using two markers on the subjects shoulders, as shown in fig. 6, bottom. The shoulder orientation represents a good simple approximation to the overall orientation of the subject. The subjects were equipped with additional markers, some of which were used to distinguish the two shoulder markers and this correctly identify the forward direction. Since we are interested in measuring the average development of positions and orientations of the subjects, we had to eliminate the natural relative oscillations that occur during a step in forward, sideward and rotational directions. For this, the collected data was filtered to eliminate the step frequency oscillations.

In the experiments we could observe stereotypic behavior of the ten subjects for most of the recorded trajectories. Another publication describing the experiments in more detail and providing a detailed statistical analysis of the collected data is currently in preparation.

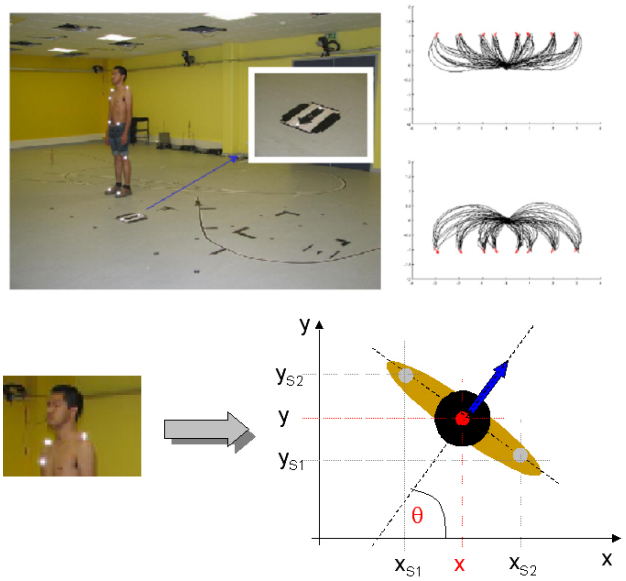

Fig. 6 Motion capture experiments on human locomotion trajectories. Global position and orientation histories of the subjects are determined using markers on the shoulders.

3.2 An optimal control model of the human locomotion path

In this section, we present the general formulation of human locomotion as an optimal control model. We give a formulation of the overall locomotion path for rest-to-rest locomotion by differential equations, as well as of a parameterized objective function, using a reasonable set of base functions. The purpose of this optimal control model is not to describe locomotion up to the last detail, but to provide a good description of the essential locomotion objectives.

In this model, we use variables x, y and θ to describe position and orientation of the locomotor system in the global reference frame. For velocities and accelerations, we shift to the human-centered reference frame, since humans do not perceive their movement in a general fixed coordinate system but rather in a local body reference frame. In this system, we can distinguish translational velocities in forward and sideward - called orthogonal - direction, v_{forw} and v_{orth}, as well as rotational velocity ω, and corresponding accelerations which are used as inputs variables u of the optimal control model $u = (u_1, u_2, u_3)^T = (a_{forw}, a_{rot}, a_{orth})^T$.

As described in the introduction, as far as translational motions are concerned, humans in most cases prefer to move in forward direction, and the orthogonal component is zero. Such a motion is called nonholonomic. However in certain situations, orthogonal velocity components appear and locomotion becomes holonomic. For

the locomotion trajectories to close-by targets studied here, we expect to observe holonomic motions or at least motion phases.

We therefore use the fully holonomic locomotion model:

$$\begin{aligned} \dot{x} &= cos\theta\, v_{forw} - \sin\theta\, v_{orth} \\ \dot{y} &= sin\theta\, v_{forw} + \cos\theta\, v_{orth} \\ \dot{\theta} &= \omega \\ \dot{v}_{forw} &= u_1 \\ \dot{\omega} &= u_2 \\ \dot{v}_{orth} &= u_3 \end{aligned} \tag{9}$$

which still contains nonholonomic motions as a special case for $v_{orth} \equiv 0$, i.e. $u_3 \equiv 0$ and $v_{orth}(0) = 0$.

The choice of base functions for the objective function was guided by some intuitive ideas: It is clear that the total time of the path has to be a free variable of the problem and humans will generally prefer faster over slower paths, i.e. tend to minimize total time. Without sudden events, humans tend to perform smooth paths, i.e. large variations of all velocities are avoided, which corresponds to a minimization of accelerations (by magnitude). Motions in forward, orthogonal and rotational direction are clearly judged differently from the subject's perspective and therefore need individual weights. This results in the following basic formulation of the objective function as a combined weighted minimization of total time and the integrated squares of the three acceleration components:

$$\begin{aligned} \Phi(T, x(t), u(t), p) &= \sum_{i=0}^{3} \left[\alpha_i \int_0^T \phi_i(x(t), u(t)) dt \right] \\ &= \alpha_0 \cdot T + \alpha_1 \int_0^T u_1^2\, dt + \alpha_2 \int_0^T u_2^2\, dt + \alpha_3 \int_0^T u_3^2\, dt \end{aligned} \tag{10}$$

The objective function is therefore composed of four base functions and has four corresponding weight parameters. In contrast to [Mombaur et al (2008)], where the parameters were determined by manual tuning for a humanoid robot model, we will here use inverse optimal control to properly identify the size of the parameters from human locomotion data. Additionally, in contrast to the qualitative robot model proposed previously, we will show in this paper, that it is not necessary to each time adjust the parameter α_3 according to the distance and orientation change of the target. We will show that is possible to approximate the human behavior in the whole area of close-by targets investigated in the experiments by a unique set of parameters $\alpha_0 - \alpha_3$. The model weights will change for far away targets and long motion segments without any rest position where the motion in general is nonholonomic. So instead of the the continuous model proposed in [Mombaur et al (2008)] we identify here a model which only requires a split into few domains.

3.3 Computational results: Identification of the objectives of human locomotion

In this section we present computational results of applying inverse optimal control to identify the objective function of problem (4) to match the human locomotion trajectories described in section 3.1. We present numerical evidence to support the hypothesis that locomotion objectives can be approximated by a simple unique model in all of the domain we investigated experimentally.

Concerning the choice of trajectories or trajectory combinations there is of course a wide range of possibilities, due to the large amount of data collected. In this paper we show how five randomly selected locomotion scenarios (a "scenario" is characterized by a target position and orientation) can very well be approximated simultaneously by the same optimal control model, i.e. the same objective function. For each scenario, we use experimental trajectories of five subjects, so the fit was performed over a total of 25 trajectories.

The variable vector z in the bilevel inverse optimal control problem formulation has dimension three: time histories of x, y and θ (i.e. three of the six state variables) are approximated at the same time. Neither velocities (the three remaining state variables) nor accelerations (the three control variables) are directly measured.

As optimal values for the objective function parameters we identified $\alpha^T = (1, 1.139, 0.159, 2.681)$, where α_0 was the parameter fixed a priori. The objective function (4) therefore becomes

$$\Phi(T, x(t), u(t), p) = T + 1.139 \int_0^T u_1^2 \, dt + 0.159 \int_0^T u_2^2 \, dt + 2.681 \int_0^T u_3^2 \, dt. \quad (11)$$

The weight factor corresponding to the orthogonal direction is about 2.5 the weight factor of the forward direction which leads to a clear preference of forward walking, but leaves the possibility for orthogonal motions whenever they are more efficient in this measure. The weight factor of the rotational term is quite small, i.e. large accelerations in rotational direction are less punished.

The top left part of fig. 7 shows the five arbitrarily chosen scenarios. The other parts of the figure show the results of simultaneous inverse optimal control for all five cases. The red solid line in all sub-figures represents the respective computed optimal trajectory for the identified set of objective function parameters. The five dashed lines denote the measured trajectories of the five subjects used as bases for the computation. The fit in all cases is very good, taking into account that the model equations and optimization functions are always a simplification, and that no perfect fit can be achieved.

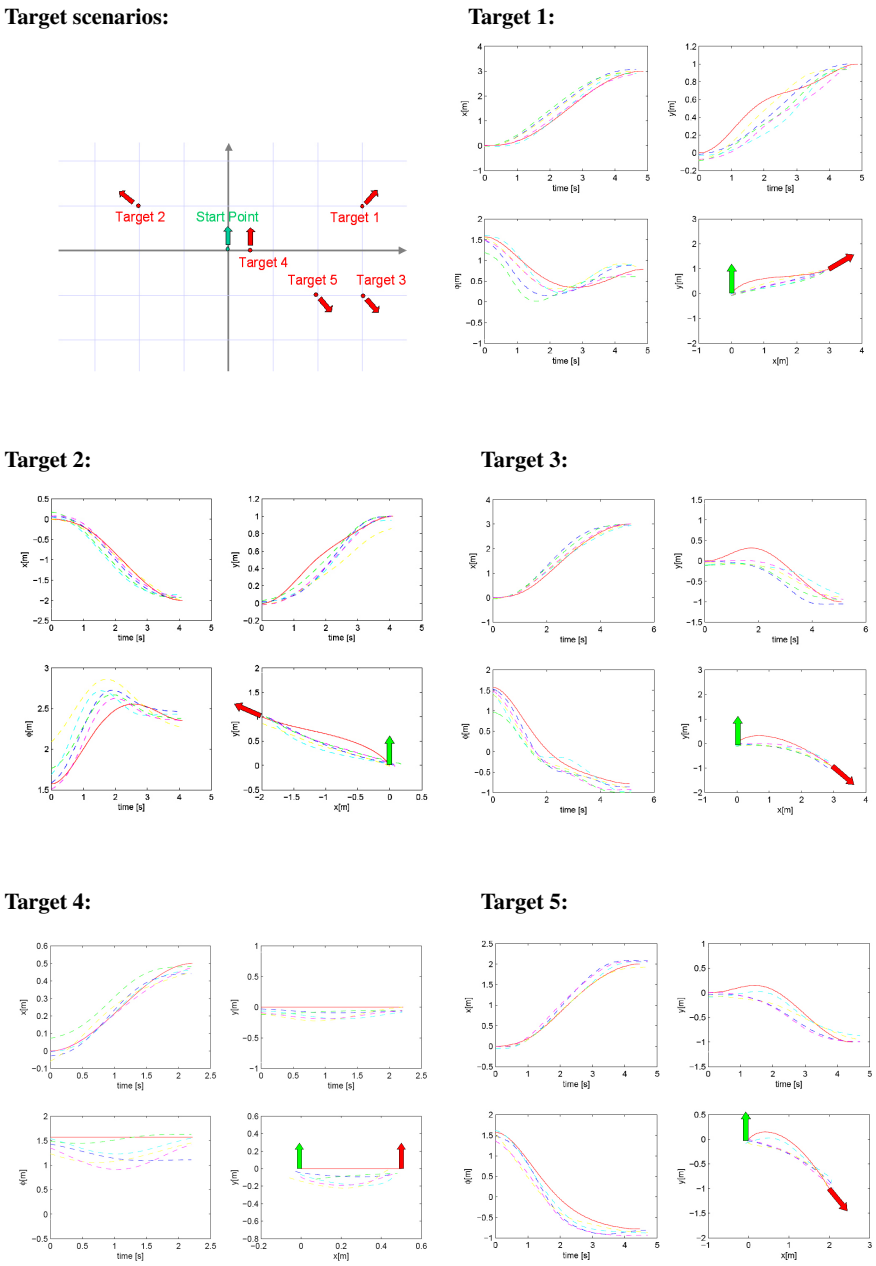

Fig. 7 Results of inverse optimal control performed simultaneously for five target scenarios. The top left sub-figure presents the five arbitrarily selected scenarios. The other sub-figures show the fit between the measurements (5 dashed lines in each case, representing 5 different subjects) with the respective optimal trajectory (solid red line) produced by the objective function identified by inverse optimal control.

3.4 Using inverse optimal control results to control humanoid robots

In this section we briefly describe how the optimal control model that has been established by inverse optimal control can be used to enable the humanoid robot HRP-2 [Kaneko et al (2004)] at LAAS to autonomously generate locomotion trajectories. As described previously, the focus of the presented research is on the generation of bio-inspired overall locomotion trajectories, i.e. the appropriate choice of the trace of the robot on the floor. Our interest here is neither the selection of foot patterns about the path nor the generation of trajectories of all internal joints. For this purpose we rely on existing approaches for the robot HRP-2.

For any given locomotion target to be reached by the humanoid robot, it is now possible to solve the optimal control problem (1) - (3) with the objective function established above in section 3.3. In the optimal control problem formulation, velocity and acceleration bounds are modified to correctly describe the limits of the humanoid robot. The solution of this optimal control problem gives the natural overall path to be followed to the target.

Linear and angular velocities of this computed path are then passed to the pattern generator. We use the walking pattern generator by Kajita et al. [Kajita et al (2003)], which is based on preview control of zero moment point (ZMP) using the table-cart inverted pendulum model, and which produces appropriate footprints and generates a desired ZMP trajectory. Leg joint angles are computed by inverse kinematics from the CoM trajectory and the footprints. The resulting biped walking motion is dynamically stable in the ZMP sense. Fig. 8 shows a visualization of the resulting robot motion for one example, using the humanoid simulator and controller software OpenHRP [Kanehiro et al (2004)] for the humanoid robot HRP-2. Due to identical interfaces of OpenHRP towards simulation and the real robot, the same motions can easily be transferred to the robot.

The computations described above have so far been performed offline. But since computation times are very short - compared to typical delays of humanoid robots - these routines could easily be implemented on the robot and and could be called each time the robot has to autonomously decide about a locomotion trajectory.

4 Conclusion & future research

The main purpose of this paper was to present inverse optimal control as a very useful approach to identify underlying optimization objectives of natural processes such as biological motions from experimental data. We have described a flexible numerical approach which allows the solution of inverse optimal control problems for very general problems.

The second purpose of this paper was to propose an optimal control model to describe human locomotion in rest-to-rest motions to close targets. Establishing this

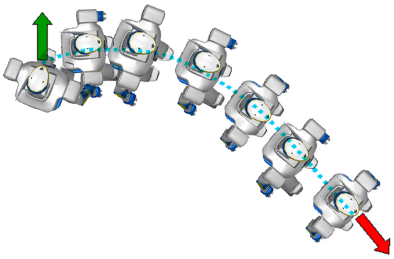

Fig. 8 Implementation of natural locomotion trajectory for target 5 on the humanoid robot HRP-2

simple and unique model was only possible using inverse optimal control. According to our computations it seems to represent a good approximation of the collected locomotion data, and it is very useful to produce natural motions of a humanoid robot.

In any inverse optimal control problem formulation, the selection of appropriate base functions for the objective function is obviously a very crucial element, since any solution of the inverse optimal control problem can only become as good as its base functions permit. For the locomotion study in this paper, we have used a simple objective function based on four elementary functions minimizing total time and accelerations. According to our results this objective function, with properly identified weight factors, seems to be able to explain much of the observed behavior. It can be expected that there are additional components of minor importance which could however slightly improve the fit. Example base functions that we plan to further investigate are terms related to the respective jerks, to the velocity components, or to energy or variation of energy of the motion. We also will establish the model for far away goals for which previous experience has already shown that the resulting motion is mainly nonholonomic. In addition, we are currently extending our research towards the study of locomotion in the presence of fixed and moving obstacles.

The generality of the presented inverse optimal control approach allows its application to a variety of other problems, such as the identification of optimization criteria of locomotion on the joint level. Based on our previous work on forward optimal control of human-like running motions [Schultz and Mombaur (2008)] [Mombaur (2008)] and the multi-body system models developed in this research, we are currently applying inverse optimal control to identify objective functions of different human running motions based on motion capture data.

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